Computational vademecums for PDEs involving parameterized geometries: towards interactive shape optimization tools

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ABSTRACT

Numerical strategies for shape optimization rely on an iterative loop in which (i) a state problem is solved to simulate the physical phenomena of interest in the current geometrical configuration; (ii) an adjoint problem is solved to compute shape sensitivities and identify a descent direction; (iii) the current shape is updated accordingly. Within this context, the main computational bottleneck is represented by the necessity of solving multiple queries of the same partial differential equation (PDE) at each iteration of the optimization loop. This is especially critical in an industrial environment in which complex geometrical models are employed and large sets of admissible shapes need to be explored. In this talk, a strategy to reduce this computational burden is presented. The method relies on an *a priori* reduced order model (ROM), i.e. the proper generalized decomposition (PGD) [1, 2], and on some recent developments on hybrid discretization techniques, namely the high-order hybridizable discontinuous Galerkin (HDG) method [3, 4] and its lowest-order version, the face-centered finite volume (FCFV) method [5, 6]. The PGD rationale is exploited to construct a surrogate model in which the solution of the PDE under analysis depends explicitly on the parameters describing the shape of the domain. The resulting ROM features (i) an offline phase in which a *computational vademecum*, i.e. a generalized solution of the high-dimensional PDE as a function of spatial and parametric coordinates, is constructed; (ii) an online phase in which the generalized solution is particularized for a specific set of parameters describing the shape of the domain. Numerical examples of the capability of the discussed strategy to compute fast and accurate approximations of the PDE under analysis for different geometrical configurations of interest will be presented, showing its potential to develop interactive shape optimization tools by reducing the cost of solving state and adjoint problems at each iteration of the optimization loop.

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