Weighted quadrature algorithm for hierarchical bases in isogeometric analysis

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ABSTRACT

In this work we propose a new algorithm for the fast computation of matrices arising from Galerkin isogeometric methods where the bases used to discretize the problem are hierarchical.

In recent years a lot of effort has been put in the development of algorithms for the efficient computation of the discretized operators arising from Galerkin isogeometric methods: from the standard *element-based approach* (a la Finite Element) described in [1] to the new paradigm described by the *weighted quadrature approach* [2, 3]. Both approaches, as described in the aforementioned references, are directly usable only for spaces having a tensor-product structure. On the other hand, hierarchical isogeometric spaces [4] are very attractive in the context of IGA for adaptive methods [5]. These spaces enjoy a tensor-product construction at the local instead of global level.

This work extends the work in [2] in order to design an efficient algorithm for computing Galerkin matrices using the weighted quadrature approach for hierarchical spaces.

The proposed algorithm has a computational cost that is proportional to the number of degrees of freedom and a favorable dependence on the polynomial degree. Numerical tests will be provided to illustrate the performance of this approach.

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