Lattice regularization and reconstruction for the deshomogenization method

O. Pantz*, P. Geoffroy-Donders† and G. Allaire†

* Laboratoire Jean Alexandre Dieudonné (LJAD)
Université Côte d'Azur
06108 Nice Cedex 02, France
e-mail: pantz@univ-cotedazur.fr, web page: https://math.unice.fr/∼pantz

† Centre de Mathmatiques APpliquées (CMAP)
École Polytechnique
Route de Saclay, 91128 Palaiseau, France
e-mail: allaire@polytechnique.edu - Web page: http://www.cmap.polytechnique.fr/∼allaire

†† Procédés et ingenierie en mécanique et matériau (PIMM)
ENSAM
151 Bd de l'Hôpital, 75013 PARIS, France

ABSTRACT

Topology optimization aims to automatically compute the best possible shape of a structure for a given purpose. It is nowadays a well developed field counting a wide variety of different techniques : Level-set, topological derivation or SIMP. Among them, the homogenization method, if appealing from the theoretical point of view, is far less popular. The main idea behind this approach consists in extending the admissible shapes to the set of composites. However a post-treatment phase is required to end up with a workable black and white design. This is traditionally done by penalizing the intermediate densities. The final result doesn't display any significant advantages over the more simple and straightforward SIMP method. Thus, to this day, there was little to support the former against the latter. Nevertheless, the emergence of mature additive manufacturing technologies, with its abilities to generate micro-structures, may change this picture. The homogenization method is indeed the perfect tool to optimize those micro-structures. Moreover, a recent post-treatment approach, called deshomogenization allows for skipping the penalization stage and to obtain black and white almost optimal designs.

The deshomogenization method provides a sequence of genuine shapes converging toward the composite optimal obtained through the homogenization method. It has been introduced in [1, 2] in a two dimensional setting and has been recently extended to the three dimensional case [3]. Slightly alternative strategies have also been developed in [4, 5].

The composite shape computed via the homogenization method is locally described as a periodic micro-structure and a given rotation. To build the sought sequence of shapes converging toward the optimal composite, a so-called grid map has to be computed. Its aims is to take into account the fact that the rotation field of the periodic cells is not constant over the optimization domain. The grid map is defined so that the inverse of its gradient is proportional to the local rotation of the cells. Nevertheless, due to invariance properties of the underlying lattice by some symmetry group, this rotation is not unequivocally defined but only up to the multiplication by an element of the symmetry group. This adds a technical difficulty when not coherent orientation can be defined. In a simply connected domain, if the rotation field is regular enough, any rotation field of the cells can be coherently oriented. But in the general case, the optimization domain is not necessarily simply connected and the rotation field can present point-wise singularities. In consequence, the basic deshomogenization method can not be applied.

In this talk, we are going to present how the deshomogenization method can be extended to handle all cases, whether the domain is not simply connected or the rotation field does contain singularities. After recalling the basic ideas behind the homogenization method (see P. Geoffroy talk for more details), we will present the generalized deshomogenization method. The principle remains exactly the same, except that we have to work on a manifold (instead of an open subset of $\mathbb{R}^n$) and have to introduce singularity functions to account for the non simple connectivity of the introduced manifold. Moreover,

a regularization step, inspired by the Ginzburg-Landau energy, is proposed to get rid of the possible unnecessary singularities of the orientation field.

Some examples to compliance optimization will be given in dimension two and three.

REFERENCES

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