# Safety assessment of historic masonry structures by Limit Analysis and partial safety factors

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## ABSTRACT

Safety assessment of historic masonry structures is a complex problem mainly due to the mechanical characteristics of their material. In 1953 Kooharian [1] showed that Limit Analysis is suitable for that type of structures and has proven effectiveness for simplified assessment [2] as long as sliding collapse does not occur . This Standard Limit Analysis is many times formulated as an optimization problem in which the bounds of the load factor is intended to calculate the maximum or minimum value with which the collapse begins. It is generally assumed that a load factor lower than the onset of collapse is a safe load factor but this assumption is false. The collapse due to the lack of stability may occur by increase or decrease of the load factor.

This work presents an alternative to load factor determination to evaluate the safety of masonry structures. The possibility to incorporate one or more safety coefficients is presented applying a partial safety factor method similar to the one proposed by Ditlevsen [3]. An important difficulty of this purpose is that these partial coefficients are applied to variables that are referred to the origin of coordinates. This would be appropriate for materials with similar mechanical behaviour under tension and compression stresses, but it is not the case for the typical materials employed in masonry structures like stones, bricks or similar. Materials with non-symmetric tension-compression behaviour have the origin of coordinates over the yield surface or very close to it. For this reason, the origin can hardly be considered as a safe reference point.

The method proposed in this work consists of the calculation of the interior point further of the yield surface [4] and considers it as the safest point. Considering that point as the origin of coordinates, the partial safety factors can be calculated. Mathematically, the safety factor $\gamma$ is obtained with the distances from the load case that is evaluated to the yield surface and to the origin, according to the expression $\gamma=y_c/(y_c-y_p)$. The value of $y_p$ determines the distance between the load case and the nearest point of the yield surface. The value of $y_c$ determines the distance from the origin to the tangent of the yield surface. According to that expression, a structure with a safety factor $\gamma=1$ means that the point of the load case is just over the yield surface and the collapse is imminent. Alternatively, different collapse modes can be considered and, from them, different partial safety factors can be calculated.

This work presents examples in which the conventional load factor is not safe in some cases, and defines the proposed method based in partial safety factors, and describes a possible implementation.

## REFERENCES

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