# The safety of masonry arches subject to vertical and horizontal forces. A numerical method based on the thrust line closest to the geometrical axis

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## ABSTRACT

In this paper the topic of the safety assessment of masonry arches based upon their geometry is investigated. The theoretical background is the Heymanian master safe theorem along with the no-tension assumption of masonry [1].

The continuous arch is analyzed considering a discrete pattern of vertical loads, such as those of the self-weight, and inertial horizontal forces, such as those provoked by an earthquake.

Among all the lines of thrust contained within the profile of the arch, the one closest to the geometrical axis can be considered the best one thanks to the minimum bending moment present in each cross section. A numerical procedure for computing the line of thrust closest to the geometrical axis of an arch subject to its self-weight has been recently formulated by the authors [2,3]. This procedure accounts for this line of thrust by minimizing the distances between the geometrical axis of the arch and the thrust line.

In order to consider the action of both vertical loads and horizontal forces proportional to the vertical ones, an extension of the procedure is herein presented, that exploits the analogy between an inclined plane and the inclination of the resultant action of the vertical and horizontal loads. According to this analogy, the effect of an horizontal load acting together with a vertical one can be accounted for by rotating the structure at the same angle as that which corresponds to the action line of the resultant.

In accordance with Heyman, collapse in masonry arches is not generally provoked by material failure because stresses are always rather small. Therefore, the safety can be assessed based on the comparison between the shape and position of the thrust line and the profile of the arch. In this context, the Heymanian geometrical factor of safety has been revisited and the full-range factor of safety has been proposed. To compute this factor the domain of equilibrium thrust lines within the profile of the arch, which is computed by shifting the thrust line upwards and downwards until it touches the extrados and intrados of the arch, is identified.

The paper discusses both the steps of the numerical procedure for computing the line of thrust through the finite difference method and the proposed factor of safety. Finally, arches of different profiles are analyzed in order to explain how the method works and to demonstrate the versatility and effectiveness of the procedure.

## REFERENCES

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