General thrust surface of the masonry domes

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ABSTRACT

Masonry domes are shell-like structures with a no-tension type material behaviour [1]. The dome geometry, material behaviour and the type of the loading define how the dome balances the load. It is known and proved that the dome could balance the load only by forces, without bending moment but cracks may appear since the material does not resist tension. The surface where the balancing forces are acting is called *the thrust surface*. If the forces are acting in the tangent plane of the thrust surface than the thrust surface is called catenary type membrane surface. It is analogous to the envelope of resultant forces in case of masonry arches. If the membrane (thrust) surface coincides with the mid-surface of the dome, then the dome is a *funicular-type shell*, i.e. a membrane shell. Otherwise, the masonry dome is safe if the thrust surface is located within the thickness of the dome, nevertheless cracks might form.

The thrust surfaces, mentioned above, can be examined and determined by e.g. thrust network analysis (TNA) [2] or by solving Pucher's equation for membrane shells, that provides the corresponding stress function [3, 4]. If the dome cracks, then the thrust surface may still be approximated by the series of thrust lines that correspond to the modified model of the cracked dome, i.e. a series of orange-slice arches [1].

It is possible to find a moment-free surface (i.e. thrust surface) to arbitrary dome geometry and loads. However, depending on the stereotomy of the dome, it is not necessarily a membrane surface. This situation is analogous to the general relation between the resultant force vector and the thrust line in the masonry arch [5]. The term *general thrust surface* is introduced to label such a moment-free surface. The present paper introduces a new method to determine the general thrust surface for a given dome geometry and stereotomy. The load is defined relative to the mid surface of the dome. A suitable reference surface is fixed for the calculation. It can be regarded as a generalized version of the base plane which is the reference surface to the Pucher's equation of the membrane shells. The projection of the load and the internal forces to the reference surface is carried out by using the masonry joint positions i.e. using the stereotomy function. Thus the general thrust surface depends on the stereotomy of the dome. Despite the rich literature investigating the phenomena's 2D equivalent (i.e. effect of stereotomy on the thrust line of masonry arches), to the authors knowledge the presented method is a novel approach in the analysis of domes. It is shown, that both the catenary-type and the funicular-type membrane surface are special cases of the general thrust surface. No-tension material behaviour and cracks in particular lead to a general thrust surface. Numerical examples illustrate the usefulness and effectiveness of the proposed method to determine the general thrust surface of a dome with a given geometry and stereotomy.

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