Safe estimation of minimum thickness of circular masonry arches considering stereotomy and different rotational failure modes

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ABSTRACT

Limit state analysis of masonry arches sets to assess the safety of the structure by determining the minimum thickness that just contains a thrust line, which, and by extension the resulting minimum thickness, is not unique for given arch geometry and loading: it is also subject to stereotomy [1]. Present paper follows the equilibrium method of the kinematic approach and adopts the Heymanian assumptions [2] regarding material qualities. The equilibrium method relies on the *a-priori* definition of kinematically admissible failure modes and determines the corresponding minimum thickness still resulting equilibrium. In this study, circular arches of constant thickness subject to self-load are considered. Their eccentricity (from the axes of the semi-circle) are parametrized by α_t, α_s, describing their 'pointiness' and opening angle, respectively. Due to the assumption of infinite friction, only rotational failure modes are considered. The paper identifies five different types (subject to geometry) in agreement with previous findings of the literature [3], labelled, based on the number of concurrent hinges, as 5-h1, 5-h2a, 5h-2b, 6-h and 7-h (see the *Figure*). The latter is analytically proven to be the maximum number of concurrent hinges for circular arches, if $\alpha_s \leq \pi/2$. Stereotomy of a standing structure is not always known, hence it is relevant to seek a stereotomy related bounding value of minimum thickness for each of the various failure modes. The analysis was simplified if there would be one distinguished thrust line (i.e. stereotomy), resulting bounding values for all cases. The envelope of resultants (linked to vertical stereotomy) results upper bound minimum thickness and hence provides a safe estimation for the semi-circular arch (5-h1 type) – therefor it seems a natural candidate.

The ranges of stereotomy related minimum thicknesses are defined for circular arches (see a section of the solution space in the *Figure*). It is shown, that the envelope of resultants does not generally result a bounding value minimum thickness, only in the case of 5-h1 and 5-h2b modes. Moreover, it leads to a lower bound in the latter case – an unsafe estimation. The paper derives both upper and lower bounds for the other failure modes and provides the necessary stereotomies leading to them. It is also explicitly shown, that stereotomy not only affects the minimum thickness value, but the corresponding failure mode as well (for certain arch geometries). Findings of the paper support the theoretical relevance of stereotomy-related studies of the stability of masonry arches.

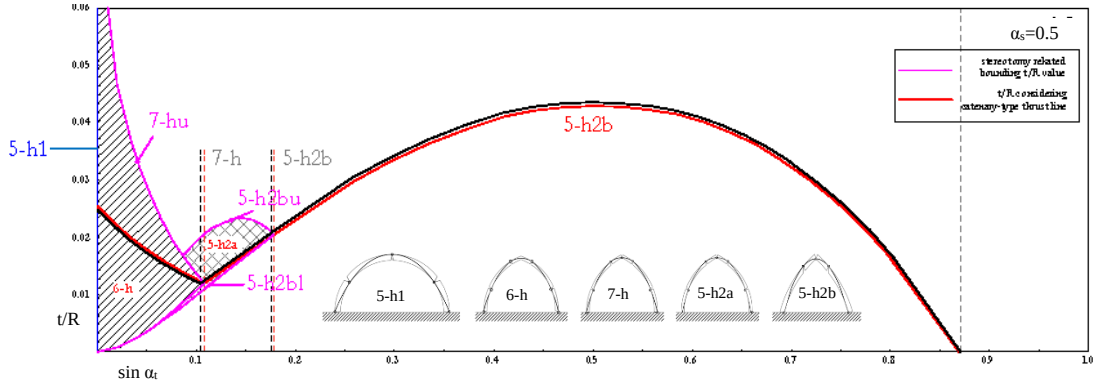

Figure: Range of minimum thickness values *(t/R)* and corresponding failure modes for circular arches with respect to α_t at fixed α_s=0.5, subject to stereotomy

REFERENCES

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