A new strategy for finding three-dimensional, lower-bound thrust networks for masonry vaults

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ABSTRACT

Complex unreinforced masonry vaults present a challenge to engineers and scientists aiming to assess its stability and safety. Heyman [1] established the principles of limit analysis in masonry structures allowing the computation of lower-bound solutions to the assessment problem by finding a network of forces in equilibrium with the masonry self-weight contained between the intrados and extrados of the real structure. Traditionally, three-dimensional geometries are simplified by slicing techniques allowing the thrust line equilibrium to be found for each slice and later combined until hitting the supports. Currently, intuitive network-based methods emerged [2] and the computation of fully three-dimension equilibrium solutions became faster and automated. These methods proved to be very efficient to the generation of unconstrained networks, suggesting its application to the design of new structures. However, the problem of assessment in historical constructions involves strong constraints that need special attention such as upper and lower bounds for the solutions, specific support positions, limited reaction forces and/or directions and the presence of hinge lines, cracks and openings.

This paper addresses this necessity by presenting a strategy for finding three-dimensional, lower-bound thrust network solutions for masonry vaults based on the manipulation of networks in equilibrium in two stages. The first stage is built on a computationally light gradient-based best-fitting optimisation algorithm that finds the closest possible solution to a previously defined target surface [3]. The distance is evaluated in each step by the calculation of the least-squares difference of the network's nodal heights to the equivalent height in the target surface. The outcome of this first stage may or may not be inside the intrados and extrados of the structure. The second stage is built on a robust optimisation algorithm based on the concept of independent edges that represent the states of self-stress of the network [4]. This stage takes an initial equilibrated network as a starting point and optimises the distribution of forces in the network such that a specific objective is achieved. The objective may be the minimisation or maximisation of the vault's reactions, the specific replication of observed cracks or even establish bounds to certain edges forces. The second stage is computationally heavier and may take part in observed symmetry or simplification of the network's topology. The strategy is validated in a real assessment case-study to prove the interest of the methodology in practical applications.

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