Structural analysis of historical constructions by graphic methodologies based on funicular and projective geometry.

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ABSTRACT

We present a graphical method for the structural analysis of domes and other surfaces of revolution, based in *Thrust-Network Analysis* presented in [1], incorporating other notions from projective geometry to make a more comprehensive method for the analysis of certain classes of structure.

In [1], the forces in a spatial network are solved by graphical analysis of the horizontal projection of the spatial network: equilibrium of the spatial system is preserved in the plane system. The forces are analysed within a dual system in which the forces in equilibrium at each joint are represented by a closed polygon.

In the following, we consider a dome as a network of lines of latitude and longitude, and analyze the equilibrium of this network in both horizontal and vertical projection. The resulting dual configuration is also a spatial system that can be considered by its projection in a horizontal and a vertical plane. This procedure not only allows the solution of the forces in a system for a given configuration, but also allows an inverse solution of the internal forces that result in a given shape.

We include a parametric study of hypothetical domes and the application a real brick hemispherical dome. The dome is divided by latitude and longitude into an arbitrary number of sectors. Equilibrium can be enforced at each node, that is, at each intersection of latitude and longitude lines. In keeping with [2], [3] and [4], and other authors, we note that the tangential forces can be considered for their net effect at each node; the net effect of two tangential forces, equal in magnitude, at a node is a radially directed force in the plane of the line of latitude: acting outwards (compression) or inwards (tension).

By considering their horizontal projection, and its dual form, it is possible to choose the shape of the radial force diagram (the vertical projection of the force diagram), and identify the radial forces associated with it, and thus the tangential forces. By an inverse method, we can determine a funicular polygon that follows the shape of the dome exactly and has a vanishing horizontal force at the apex of the dome.

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