A unifying formulation for the lower-bound analysis of systems of masonry arches and buttresses

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ABSTRACT

Since the first discovery by Robert Hook of the analogy between the structural behavior of a masonry arch with that of a catenary, for approximately three centuries the analysis of masonry structures has been pursued through the research of equilibrium states under compressive forces. On the one hand, Heyman's formulation of the lower-bound limit analysis theorem for masonry arches provided a theoretically sound framework for that ancient engineering practice [1]. On the other hand, the essential analysis tools for the application of that theorem remained funicular models and graphic statics. Conversely, a different strategy was undertaken by Milankovitch, who at the beginning of the 20th century formally derived the equilibrium equations of a circular arch and obtained a closed-expression for minimum thrust-line [2].

In last decades, modern computers and advanced numerical methods have made possible to translate those principles in powerful tools for the analysis of masonry constructions. Among them, it is worth mentioning rigid block computational analysis, originated from the pioneering work by Livesley [3]; computational thrust-line analysis [4], implementing classical graphic constructions in a computer program; approaches based on the thrust network analysis [5]; numerical solution strategies of generalized Milankovitch equations [6].

In the present work, a unifying formulation is presented for the lower-bound analysis of general monodimensional masonry elements, such as arches and buttresses, with arbitrary stereotomy. From the theoretical standpoint, the capability of the present formulation to encompass the peculiar features of the aforementioned approaches is proven. From the computational standpoint, numerical simulations are presented to show simplicity, flexibility, accuracy and robustness in the lower-bound analysis of masonry structures.

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