

Scaling laws for implicit viscosities in smoothed particle hydrodynamics

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ABSTRACT

Since its conception in 1977 [1] the SPH method has been used in a wide range of scientific and industrial applications due to its simplicity to implement and its flexibility in modelling (multi-phase) fluids with free surfaces [2, 3]. Despite the successful application of the method, it is still lacking a theoretical proof of its convergence with respect to the spatial discretization, i.e., the SPH particle size.

Several studies focused on the accuracy of SPH shear flow results. Posch et al. [4] described implicit SPH viscosities, i.e., artificial shear stress contributions which have their origin in the particle-based nature of the method. Ellero et al. [5] investigated this phenomenon further and quantified a kinetic contribution which acts as an average Reynolds stress and a potential contribution without continuum analog. Both contributions can become larger than the specified shear viscosity of the system for low viscous flows. Previous work of the present authors [6] showed that the magnitudes of the implicit viscosities depend on the used SPH formulation of the Navier-Stokes viscosity term. Imaeda et al. [7] reported large density errors which were attributed to the local shear deformation of the neighborhood of each SPH particle.

In the present work we analyze the resulting density, pressure and shear stress of SPH shear flow simulations from a different perspective, i.e., by interpreting SPH particles as grains in dense granular flow. This approach enables the mapping of several relations which are valid for granular rheology to SPH simulations and, thus, a detailed quantification of the implicit viscosities.

It is demonstrated that deviations of pressure and shear stress in SPH shear flow simulations from their theoretical values can be expressed as functions of dimensionless system parameters, namely a Reynolds number (Re), a Mach number (Ma) and an inertial number (I).

The identified scaling relations render quantitative predictions of the accuracy of the stress in SPH simulations possible. Moreover, the influence of the spatial SPH discretization, i.e., the particle size, on the accuracy can be quantified.

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