NONLINEAR STABILITY OF THE MPM METHOD

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ABSTRACT

The Material Point Method (MPM) may be viewed as being a solid mechanics method that is derived from the fluid implicit particle, FLIP and PIC methods and which has had considerable success on large deformation problems. Despite this success many theoretical issues to do with MPM remain unresolved. One such issue is the stability of the method, given its nonlinear nature. Currently either a a fourier-based analysis [1], [3] or energy-conservation approach [2] is taken. However [7] rightly point out that the nonlinear nature of the MPM scheme makes classic linear stability analysis inappropriate. Similarly while energy conservation is of great importance it does not necessarily imply stability [5]. One way to start to address this is to note that the standard time integration methods used in MPM coresponds to the use of the semi-implicit Euler method, or symplectic Euler-A [4]. There is convergence and stability analysis of this method in [6] and this analysis is sufficiently general to be applied to the MPM, providing that care is taken with the nonlinear nature of MPM. The intention here is to use this approach to shed some light on the nonlinear stability of MPM by considering a one dimensional model problem as an ordinary differential equations system in the values at particles and nodes. The aim is to consider how to bound the timestep when nonlinearity is taken into account. The stability analysis of [6] is adapted to MPM and used to derive a stable timestep bound for a model problem. This bound is contrasted against a traditional CFL bound.

REFERENCES

[1] S.~Bardenhagen, Energy conservation error in the material point method for solid mechanics, *J. of Comput. Physics*, 180, 2002,383-403.

[2] J.~Brackbill, The ringing instability in particle-in-cell calculations of low-speed flow, *J. of Computat Physics*, 75, 1988, 469-492.

[3] M. Gong, *Improving the Material Point Method*, Ph.D. thesis, The University of New Mexico (2015). http://hdl.handle.net/1928/30386

[4] B. Leimkuhler and S. Reich, Simulating Hamiltoniam dynamics. Canbridge Monographs on Applied and Computational Science.Cambridge University Press 2004.

[5] M. Ortiz, A note on energy conservation and stability of nonlinear time-stepping algorithms. Computers and Structures 24, 1, 1986, 167-168.

[6] R. Spigler and M, Vianello, Convergence analysis of the semi-implicit Euler method for abstract evolution equations. Numer. Func. Anal. and Opt.16, 1995, 5-6, 785-803.

[7] P. C. Wallstedt and J. E. Guilkey, An evaluation of explicit time integration schemes for use with the generalized interpolation material point method. *J. Comput. Phys*. 2008, 227, 22, 9628-9642.