High-order numerical scheme for vortex sheet approximation in vortex methods for 2D flow simulation

Kseniia S. Kuzmina* and Ilia K. Marchevsky

* Bauman Moscow State Technical University (BMSTU)
Applied mathematics department
2-nd Baumanskaya st., 5, 105005, Moscow, Russia
e-mail: kuz-ksen-serg@yandex.ru, iliamarchevsky@mail.ru

ABSTRACT

The problem of developing of high-order numerical schemes is considered for solving of boundary integral equation for vortex sheet intensity, which arises in 2D meshless vortex particle methods. The modification of vortex method is used which corresponds to the integral equation of the 2-nd kind with bounded kernel (for smooth airfoils) [1]. When solving this equation in vortex methods airfoil is usually approximated with a polygon consisting of *n* rectilinear segments (panels) and solution of boundary integral equation is assumed to be piecewise-constant or piecewise-linear function. Numerical experiments show that such approach provides the 2-nd order of accuracy for potential flow problems, but when it is necessary to simulate the flow in presence of vorticity the accuracy sometimes becomes rather low [2]. It is especially important for flow simulation near the airfoil (in boundary layer). This problem can be solved by improving of airfoil camber line discretization, i.e. by taking into account its curvilinearity.

It is suggested to discretize airfoil camber line by cubic polynomials over the panels. If not only the coordinates of the polygon vertices are known, but also tangent directions at them for exact airfoil shape, it is possible to approximate it by C^1-curve which provides $O(h^4)$ error of approximation, where h is panel length.

The numerical scheme is developed for boundary integral equation discretization for such camber line approximation. Numerical solution is assumed to be piecewise linear function over the panels. The resulting linear equation system follows from Discontinuous Galerkin-type approach. The efficient way is found for computation of linear system coefficients. The most part of the corresponding double integrals in matrix coefficients can be computed numerically by using Gaussian quadratures with not more than 4 integration points; for the others semi-analytical approximate expressions are obtained.

The developed method provides the 4-th order of accuracy in L_2 norm with respect to intensity of vortex sheet for potential case. It allows to raise significantly the accuracy of flow simulation in proximity of the airfoil, what is important for viscous flows simulation around bluff airfoils when it is necessary to determine the position of flow separation zone with high precision. It should be taken into account that not only vorticity influence on the airfoil should be calculated with high accuracy, but also the inverse influence – how the vortex sheet influences the motion of vorticity in the flow.

Some model problems are considered, the results are in good agreement with theoretical estimations.

REFERENCES

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