Continuum forms and their applications in predicting dynamic properties of brick mortar systems

Sumet Supprasert+, Maddegedara L. L. Wijerathne†* and Muneo Hori†

+ Department of Civil Engineering
The University of Tokyo, Bunkyo-ku, 113-0032 Tokyo, Japan
e-mail: sumet.res@gmail.com

† Earthquake Research Institute
The University of Tokyo
Yayoi 1-1-1, Bunkyo-ku, 113-0032 Tokyo, Japan
e-mail: lalith@eri.u-tokyo.ac.jp, hori@eri.u-tokyo.ac.jp

ABSTRACT

We derived three equivalent continuum forms for regularly packed interacting particles systems undergoing infinitesimal deformation. Although discrete methods like RBSM and DEM are widely used in studying particle systems, approximate continuum forms can provide some advantages. As an example, based on the obtained continuum forms, we obtained analytic predictions for wave characteristics, implemented a finite element model to simulate brick structures and explored a possible damping mechanism due to high frequency spin. While these formulations are valid for any regularly packed linearly interacting particle systems, in this paper we consider only brick mortar systems due to possible engineering applications.

Two approaches are used in deriving continuum forms; continuumnization[1] and Taylor series expansion. Both the approaches assume; the bricks are rigid and regularly packed, rotation and relative translations to be infinitesimal and brick interactions can be modeled with infinitesimal linear springs. In addition, both the methods assume the existence of smooth and continuous vector fields $\boldsymbol{u}(\boldsymbol{x})$ and $\boldsymbol{\theta}(\boldsymbol{x})$ such that $\boldsymbol{u}^{\alpha} = \boldsymbol{u}(\boldsymbol{x}^{\boldsymbol{\alpha}})$ and $\boldsymbol{\theta}^{\alpha} = \boldsymbol{\theta}(\boldsymbol{x}^{\boldsymbol{\alpha}})$, where $(.)^{\alpha}$ are quantities related to α^{th} brick. Continuumnization obtains a first order accurate continuum form by approximating the difference operations in the discrete governing equations with suitable derivatives. On the other hand, the Taylor series approach approximates $\boldsymbol{u}^{\alpha'}$ and $\boldsymbol{\theta}^{\alpha'}$ of neighboring bricks α''s of a α^{th} brick using Taylor series expansion. A second order accurate continuum model is obtained based on second order Taylor expansion. Solving the characteristic equations of these continuum forms, relations between frequency and wavenumbers for p-, s- and rotational waves are predicted.

The above predictions are verified by comparing with the results of RBSM models. It is found that the first order solution is valid for very large wave lengths, while the second order model is valid for wavelengths greater than $7a$, where a is brick length. In addition, analytic predictions accurate for all the meaningful wave lengths $(> 2a)$ is obtained by including all the infinite terms in Taylor series. This infinite series based analytic solution can reproduce the RBSM results remarkably well including non-linear regions, and will be useful in physics based applications requiring high precision, while the former two are sufficient for engineering applications.

According to the analytical predictions and the numerical simulations, the rotational frequency of the bricks are too high to be physically feasible. These high frequency spins should decay rapidly causing the system to lose energy (i.e. damping effect). We explore this possible damping mechanism using a FEM implementation based on the above obtained first order continuum form.

REFERENCES

[1] M. Hori, L. Wijerathne, J. Chen, and T. Ichimura, "Continuumnization of regularly arranged rigid bodies", *Journal of JSCE*, **4**, 38-45, (2016).