

# On strain-deformation-fabric relationships for granular materials

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## ABSTRACT

In micromechanics of granular materials relationships are investigated between the behaviour at the macro-scale, continuum level and the micro-scale level of particles and interparticle contacts.

At the macro-scale level the deformation is characterised by the strain tensor, while at the micro-scale level the deformation is characterised by the relative displacements between particles that are in contact.

The objective of the current investigation is to establish relationships between the deformation at micro-scale and macro-scale level. This is analogous the development of the so-called stress-force-fabric relationship that expresses the (relative) shear strength at the macro-scale, continuum level in terms of the magnitudes of interparticle contact forces and of the fabric tensor at the micro-scale level. The second-order fabric tensor expresses, in a statistical manner, the micro-scale structure of interparticle contacts.

To obtain the required detailed information at the micro-scale, two-dimensional Discrete Element Method simulations of isobaric tests with disk-shaped particles have been performed. Simulations employing a dense and a loose initial system, with various values for the interparticle friction coefficients, have been considered.

From continuum-mechanical considerations, it follows that the relative displacement between particles in contact depends on the orientation of the branch vector that connects the centroids of such particles. Therefore, these relative displacements at contacts (as obtained from the Discrete Element Method simulations) are averaged over groups of contacts with the same orientation.

Clusters of particles have been identified that deform as nearly-rigid blocks. Hence, these clusters have deformation characteristics that differ significantly from that of other groups of particles.

An analytical expression is developed that gives the macro-scale, continuum dilatancy rate in terms of components of the (averaged) relative displacements at interparticle contacts and of a fabric tensor.

The developed expression for the macro-scale, continuum dilatancy rate in terms of these deformation characteristics at the micro-scale is verified using results from the Discrete Element Method simulations.