Algebraic Multigrid for the Finite Pointset Method

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ABSTRACT

Algebraic Multigrid (AMG) methods [1] are known to be robust and scalable linear solvers for a wide class of problems. The key multigrid idea is the combination of classical relaxation solvers like Jacobi or Gauss-Seidel (which can reduce oscillating error components quickly) with a hierarchy of coarser meshes such that each error frequency is handled on an adequate grid. While in geometric multigrid the coarse meshes, the inter-mesh transfer operators and the coarse grid operators must be supplied by the user, AMG methods construct the hierarchy automatically from the initial linear system matrix. Moreover, no “mesh” or “grid” is required, which makes AMG attractive for solving the linear systems arising from mesh-free discretization methods.

Still, most AMG schemes were developed with some mesh-based (finite difference, finite element or finite volume) discretization of an elliptic PDE in mind. The resulting linear systems are usually symmetric, positive definite and possess the M-matrix property (or deviate only slightly from these properties). AMG algorithms employ these properties to build the hierarchy.

In this talk, we present our AMG techniques to obtain the solution of the linear systems inside the Finite Pointset Method (FPM) [2]. FPM employs a moving point cloud to discretize the PDEs using a generalized finite difference method. The resulting systems are neither symmetric nor M-matrices, even if self-adjoint operators were discretized. We describe AMG ansatzes to solve scalar pressure systems, vectorial velocity systems and coupled velocity-pressure Stokes-like systems arising from FPM's discretization of the incompressible Navier-Stokes equations and illustrate the effectiveness of our algorithms in several numerical results.

(This is joint work with the Grid-free methods group at Fraunhofer ITWM.)

REFERENCES

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[2] J. Kuhnert, “Meshfree numerical schemes for time dependent problems in fluid and continuum mechanics”, In: Sudarshan, S.: *Advances in PDE modeling and computation*, New Delhi (2014).