

Differential Complexes in Continuum Mechanics

Arash Yavari*, Arzhang Angoshtari†

* School of Civil and Environmental Engineering &
The George W. Woodruff School of Mechanical Engineering,
Georgia Institute of Technology, Atlanta, GA 30332, USA
E-mail: arash.yavari@ce.gatech.edu

† School of Civil and Environmental Engineering,
Georgia Institute of Technology,
Atlanta, GA 30332, USA
e-mail: arzhang@gatech.edu

ABSTRACT

We present differential complexes for continuum mechanics, and in particular nonlinear elasticity, that involve both symmetric and non-symmetric second-order tensors. We show that the tensorial analogue of the standard grad-curl-div complex can simultaneously describe the kinematics and the kinetics of motions of a continuum when formulated in terms of the first Piola-Kirchhoff stress and the deformation gradient. The relation between this complex and the de Rham complex allows one to derive the necessary and sufficient conditions for the compatibility of the displacement gradient and the existence of stress functions on non-contractible bodies. We discuss similar complexes for Cauchy stress and the second Piola-Kirchhoff stress. We also derive the local compatibility equations in terms of the Green deformation tensor for motions of 2D and 3D bodies, and shells in curved ambient spaces with constant curvatures.