Monte Carlo simulation vs Spectral Galerkin method:

A numerical performance study

V. Papadopoulos*, G. Stavroulakis†, D.G. Giovanis†, M. Papadrakakis†

* National Technical University of Athens, Institute of Structural Analysis and AntiSeismic Research, Iroon Polytechniou 9, Zografou Campus, Athens 15780, Greece vpapado@central.ntua.gr

[†] National Technical University of Athens, Institute of Structural Analysis and AntiSeismic Research, Iroon Polytechniou 9, Zografou Campus, Athens 15780, Greece <u>stavroulakis@nessos.gr</u> <u>dgiov16@yahoo.gr</u> <u>mpapadra@central.ntua.gr</u>

ABSTRACT

The most straightforward technique of solving stochastic partial differential equations (PDE) is the widely applicable non-intrusive Monte Carlo (MC) methods. They can handle any type of problems (linear, nonlinear, dynamic) as well as any kind of uncertainty in the load or in the system properties. In particular, when dealing with deterministic external loading, MC methods feature the solution of successive linear systems with multiple left-hand sides, since only the coefficient matrix K changes in every simulation. On the other hand, recently proposed approach, such as spectral stochastic finite element method (SSFEM), is intrusive and is using tensor product spaces for the spatial and stochastic discretizations [1]. The augmented systems that are generated when using SSFEM are suitable candidates for iterative solvers since they are flexible enough to be custom tailored to their particular architecture of the augmented systems while they are amenable to be efficiently implemented in high performance computing environments. Solution techniques are based on Gauss-Jacobi iterative solvers for addressing this problem [2-4].

This work revisits the computational performance of non-intrusive Monte Carlo versus intrusive Galerkin methods of large-scale stochastic systems in the framework of high performance computing environments. The purpose of this work is to perform an assessment of the range of the relative superiority of these approaches with regard to a variety of stochastic parameters. In both approaches, the solution of the resulting algebraic equations is performed with a combination of primal and dual domain decomposition methods implementing speciffically tailored preconditioners. The solution of repeated simulations of the Monte Carlo method is accelerated with an A-orthogonalization procedure aiming at reducing the iterations of subsequent simulations, while the solution of the augmented equations of the stochastic Galerkin method is enhanced with preconditioners which combine the block diagonal features of the resulting matrices as well as the sparsity pattern of the off block-diagonal terms. Numerical results are presented demonstrating the efficiency of the proposed implementations on a large-scale 3D problem with different stochastic characteristics and useful conclusions are derived regarding the ranges of stochastic parameters in which non-intrusive solvers have a superior performance compared to intrusive ones and vice versa.

REFERENCES

[1] R. Ghanem and P.D. Spanos. Polynomial chaos in stochastic finite elements. J. App. Mech., ASME, pages 197-202, (1990).

[2] D. Ghosh, P. Avery, and C. Farhat. A method to solve spectral stochastic finite element problems for large-scale systems. *Int. J. Numer. Meth. Engng.*, 00: 1-6, (2008).

[3] D. Ghosh, P. Avery, and C. Farhat. A FETI-preconditioned conjugate gradient method for large-scale stochastic finite element problems. *Int. J. Numer.Meth. Engng*, 80:914-931, 2009.

[4] M.F. Pellissetti and R. Ghanem. Iterative solution of systems of linear equations arising in the context of stochastic _nite elements. *Adv. Engrg. Software*, 31:607-616, (2000).