1st Pan-American Congress on Computational Mechanics - PANACM 2015
XI Argentine Congress on Computational Mechanics - MECOM 2015
S. Idelsohn, V. Sonzogni, A. Coutinho, M. Cruchaga, A. Lew & M. Cerrolaza (Eds)

PENALTY-FINITE ELEMENT APPROXIMATIONS OF SLIP BOUNDARY CONDITIONS

JOSÉ M. URQUIZA (*) AND IBRAHIMA DIONE

GIREF, Département de mathématiques et de statistique, Université Laval
1045 avenue de la médecine, G1V 01A, Québec, Qc, Canada
jose.urquiza@mat.ulaval.ca, ibrahima.dione.1@ulaval.ca

Key words: Computational Fluid Mechanics, Slip Boundary Conditions, Penalty Method

Abstract.

The penalty method is a classical and widespread method for the numerical treatment of constrained problems such as unilateral contact problems and problems with Dirichlet boundary conditions. It allows to impose these boundary conditions weakly and is an alternative approach which avoids the introduction of additional unknowns in the form of Lagrange multipliers, for instance.

In the case of slip boundary conditions for fluid flows (or solid elastic materials) on a bounded domain with a smooth curved boundary, one of the main obstacles to its efficiency and to its mathematical analysis is that a Babuska's type paradox occurs: on a sequence of polygonal domains converging to the smooth domain, the solutions of the corresponding problems do not converge to the solution of the problem on the limit domain.

Our presentation will focus on the finite element approximation of Stokes equations with slip boundary conditions imposed with the penalty method. First, for polygonal or polyhedral boundary approximations induced by the finite element meshes, we prove convergence estimates in terms of both the penalty and discretization parameters. In particular, we show that by carefully choosing the penalty parameter in terms of the discretization parameter, Babuska's paradox can be overcome [1]. Secondly, based on numerical results, we show how different variants of the original penalty method perform in terms of convergence rates. These variants involve the smoothing of the normal vector, the use of isoparametric finite elements or reduced integration of the penalty term [2].
[1] Dione, I. and Urquiza, J.M. Penalty-finite element approximation of Stokes equations with slip boundary conditions. *Numerische Mathematik*, in press, published online (2014), DOI 10.1007/s00211-014-0646-9, 24 pages.
[2] Dione, I., Tibirna, C. and Urquiza, J.M. Stokes equations with penalized slip boundary conditions. *Int. J. Comp. Fluid. Dynam.* (2013) **27**:283–296.