

# Formulation of a macro hierarchical finite element for the analysis of thick laminated beams

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## ABSTRACT

The formulation of an enriched macro element suitable to analyze the static and dynamic behavior of composite laminated beams is presented in this article. Based on the Trigonometric Shear Deformation Theory (TSDT) [1] and refined theories [2], and the use of Gram–Schmidt orthogonal polynomials as enrichment functions, a hierarchical finite element is developed [3]. In the TSDT framework, shear stresses are vanished at the top and bottom surfaces of the beams and shear correction factors are no longer required. The Principle of Virtual Work is applied to derive the governing equations. The developed finite element is free of shear locking and the transversal shear stresses computed with the developed formulation are in good agreement with those obtained by means of three-dimensional analysis. Finally, results for static and free vibration analysis of symmetric and non-symmetric laminated beams, having different thickness to length ratios and boundary conditions are presented to show the accuracy and applicability of the present approach.

## REFERENCES

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