A two-level algorithm for the solution of heterogeneous Helmholtz problems

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ABSTRACT

The Helmholtz equation $-\Delta u - k^2 u = f$ with suitable boundary conditions and wave number $k > 0$ governs wave propagation and scattering phenomena arising in a wide range of engineering applications, such as aeronautics, underwater acoustics, and geophysical seismic imaging. Its discretization with piecewise linear finite elements results for large wave number k in an indefinite, ill-conditioned linear system of equations. The number of grid points grows rapidly with k in order to maintain accuracy and to avoid the pollution effect [1]. As the linear system of equations is hard to solve for iterative methods [2], carefully designed methods are necessary. Preconditioning with a shifted, easier problem has been considered e.g. in [3]. The problems encountered when multigrid methods are applied to the Helmholtz equation have been analyzed in detail [4, 5].

The inherently parallel domain decomposition methods constitute a promising class of preconditioners. An essential element of these methods is a good coarse space. Here, the Helmholtz equation presents a particular challenge, as even slight deviations from the optimal choice can be devastating. In this work we present a coarse space that is based on local eigenproblems involving the Dirichlet-to-Neumann operator. Our construction is completely automatic, ensuring good convergence rates without the need for parameter tuning. Moreover, it naturally respects local variations in the wave number and is hence suited also for heterogeneous Helmholtz problems. The resulting method is parallel by design and its efficiency is demonstrated on 2D homogeneous and heterogeneous numerical examples.

REFERENCES

[1] I. M. Babuska, S. A. Sauter, Is the pollution effect of the FEM avoidable for the Helmholtz equation considering high wave numbers?, SIAM Rev. 42 (2000) 451–484.

[2] O. G. Ernst, M. J. Gander, Why it is difficult to solve Helmholtz problems with classical iterative methods, in: Numerical Analysis of Multiscale Problems, LNCSE Springer, 2012, pp. 325–363.

[3] Y. A. Erlangga, C. Vuik, C. W. Oosterlee, On a class of preconditioners for solving the Helmholtz equation, Appl. Numer. Math. 50 (2004) 409–425.

[4] A. Brandt, I. Livshits, Wave-ray multigrid method for standing wave equa-tions, Electron. Trans. Numer. Anal. 6 (1997) 91.

[5] H. C. Elman, O. G. Ernst, D. P. O'Leary, A multigrid method enhanced by Krylov subspace iteration for discrete Helmholtz equations, SIAM J. Sci. Comput. 23 (2002) 1291–1315.