Direct Solution of Partial Differential Equations Using the Wavelet-Galerkin Method

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ABSTRACT

The use of multiresolution techniques and wavelets has become increasingly popular in the development of numerical schemes for the solution of partial differential equations (PDEs). Therefore, the use of wavelets as basis functions in computational analysis holds some promise due to their compact support, orthogonality, localization and multiresolution properties.

Daubechies and Deslauriers-Dubuc wavelets have been successfully used as basis functions in several schemes like the Wavelet-Galerkin Method (WGM) and the Wavelet Finite Element Method (WFEM).

Another possible advantage is the fact that the calculation of integrals of inner products of wavelet basis functions and their derivatives can be made by solving a linear system of equations, thus avoiding the problem of using approximations by some numerical method. These inner products were defined as connection coefficients and they are employed in the calculation of stiffness matrices and load vectors. In this work, all the mathematical foundations and computer implementation aspects regarding wavelet functions, their derivatives and connection coefficients are reviewed.

In this work, the Galerkin Method has been adapted for the direct solution of partial differential equations in a meshless formulation using interpolating wavelets (Interpolets). One and two-dimensional examples are proposed. The method was particularly successful in dealing with thin plates modeling.

REFERENCES

- [1] Burgos, R. B., Cetale Santos, M. A. and Silva, R. R., 2013. Deslauriers-Dubuc interpolating wavelet beam finite element. Finite Elements in Analysis and Design, vol. 75, pp. 71-77.
- [2] Deslauriers, G. and Dubuc, S., 1989. Symmetric iterative interpolation processes. Constructive Approximation, vol. 5, pp. 49-68.
- [3] Z. Shi, D. J. Kouri, G. W. Wei, and D. K. Hoffman, "Generalized symmetric interpolating wavelets", Computer Physics Communications, vol. 119, pp. 194-218, 1999.
- [4] Nguyen, V. P., Rabczuk, T., Bordas, S. and Duflot, M., 2008. Meshless methods: a review and computer implementation aspects. Mathematics and Computers in Simulation, vol. 79, pp. 763-813.
- [5] A. Latto, L. Resnikoff, and E. Tenenbaum, "The evaluation of connection coefficients of compactly supported wavelets", Proceedings of the French-USA Workshop on Wavelets and Turbulence, pp. 76-89, 1992.