

# Nonsymmetric coupling of boundary elements and ultraweak finite elements and DPG method with optimal test functions

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## ABSTRACT

We propose and analyze a numerical method to solve an elliptic transmission problem in full space. The method consists of a variational formulation involving standard boundary integral operators on the coupling interface and an ultraweak finite element formulation on the interior. To be precise, it is a nonsymmetric coupling as developed in [3], but with an ultraweak finite element part as used in [2]. To guarantee the discrete inf-sup condition, we discretize the whole system with optimal test functions. In this way, we obtain quasi-optimality of the method in the so-called energy norm. In order to relate the energy norm to usual Sobolev norms, we use recent results regarding the nonsymmetric coupling of finite elements and boundary elements on polygonal interfaces, cf. [1,4].

## REFERENCES

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