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RECENT DEVELOPMENTS FOR THE DISCONTINUOUS PETROV-GALERKIN (DPG) METHOD WITH OPTIMAL TEST FUNCTIONS

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Abstract. DPG method wears three hats [**?**]. It is a Petrov-Galerkin method with optimal test functions computed on the fly to reproduce stability properties of the continuous variational formulation [**?**]. It is also a minimum-residual method with residual measured in a dual norm corresponding to a user-defined test norm[**?**]. This gives (an indirect) possibility of controlling the norm of convergence, a feature especially attractive in context of constructing robust discretizations for singular perturbation problems [**?**, **?**].

Finally, it is also a mixed method in which one solves simultaneosly for the residual [**?**] which enables automatic adaptivity.

In practice, the optimal test functions and residual are approximated within a finite-dimensional *enriched* test space. The mixed method framework provides a natural starting point for analyzing effects of such an approximation through the construction of appropriate Fortin operators[**?**].

What makes the whole story possible is the use of discontinuous test functions (*broken test spaces*). The trial functions may but need not be dicsontinuous. The paradigm of "breaking" test functions can be applied to classical second order and mixed formulations as well as less known *ultra-weak* variational formulations[**?**]. It results in a *hybridization* of the original formulation where one solves additionally for fluxes (and traces in the ultraweak case) on the mesh skeleton. The hybridization approximately doubles the number of unknowns when compared with classical conforming elements or HDG metods but it is comparable with the number of unknowns for other DG formulations as well as mixed methods. Computation of optimal test functions and residual is done locally, at the element level. It does not contribute thus to the cost of the global solve but it is quite significant for systems of 3D equations.

The DPG methodology guarantees a stable discretization for *any well-posed* boundary- or initial boundary-value problem (space-time formulations). In particular, it can be applied to all problems where the standard Galerkin fails or is stable only in the asymptotic regime. Being a Ritz method, DPG enables adaptive computations starting with coarse meshes. For instance, for wave propagation problems, the initial mesh need not even to satisfy the Nyquist criterion.

We will illustrate the points made above by focusing the presentation on one selected application example - three dimensional Maxwell equations.

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