ON THE SECOND-ORDER ACCURACY OF VOLUME-OF-FLUID INTERFACE RECONSTRUCTION ALGORITHMS

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Abstract. Let $z(s)$ be a two times continuously differentiable curve on a bounded, simply connected domain Ω in the plane that separates two materials or fluids, say materials 1 and 2. Cover Ω with a uniform square computational grid of side h and suppose that the only information one is given about the curve z is the area fraction $0 <= \Lambda_{ij} <= 1$ that is occupied by material 1 in each cell C_{ij} of the grid. I will show how to construct an approximation to the curve z using a single line segment in each cell, which is second-order accurate in the max norm and outline a proof of this fact.

Numerical methods for approximating a curve or a surface in three dimensions on a computational grid that are based solely on the volume fraction information associated with the curve are known as volume-of-fluid (VOF) methods. The problem I have outlined above is known as the "interface reconstruction problem" for a VOF method. Besides being the first proof that an algorithm for solving this problem converges to the given interface z, this result is interesting because it provides a criterion for determining whether the reconstructed interface is "well-resolved" on a given grid. This criterion depends only on the curvature $\kappa(s)$ of the initial data $z(s)$ in the 3×3 block cells centered on the cell C_{ij}. Namely, given a square grid of side h covering a two times continuously differentiable simple closed curve $\mathbf{z}$ in the plane, one can construct a pointwise second-order accurate piecewise linear approximation $\tilde{\mathbf{z}}$ to $\mathbf{z}$ from just the volume fractions due to $\mathbf{z}$ in the grid cells. I prove a sufficient condition for $\tilde{\mathbf{z}}$ to be a second-order accurate approximation to $\mathbf{z}$ in the max norm is h must be bounded above by $2\,/\,(33\,\kappa_{max})$, where κ_{max} is the maximum magnitude of the curvature κ of $\mathbf{z}$. This constraint on h is solely in terms of an intrinsic property of the curve $\mathbf{z}$, namely κ_{max}, which is invariant under rotations and translations of the grid. An important consequence of the proof is that the max norm of the difference $\mathbf{z} - \tilde{\mathbf{z}}$ depends linearly on κ_{max}.

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