A Novel Algebraic Extra-Stress Model and its Analytic Solution for Fully Developed Laminar Flows

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ABSTRACT

Two-dimensional viscoelastic flows described by differential models employ three partial differential equations for calculating the components of the extra-stress tensor whereas an algebraic model requires only one partial differential equation to be solved. Therefore, algebraic models are attractive because their solutions demand less CPU time and the algebraic equations make the computational algorithms easier to implement.

The first algebraic extra-stress model (AESM) was developed by Mompean and Deville [1] who used the ideas for modeling turbulent flows of Newtonian fluids and derived an AESM model to obtain approximate solutions of flows described by the Oldroyd-B differential model. In that work, numerical results were performed in order to demonstrate the abilities of the formulation. In a subsequent paper, Mompean [2] extended the idea of the Oldroyd-B AESM to approximate the differential PTT model.

In this work, we use the ideas presented in the methodology proposed in [1, 2], and derive an AESM to approximate the Finite Extendable Non-linear Elastic - Chilcott and Rallison (FENE-CR) differential model. An algorithm for solving the governing equations for incompressible flows of FENE-CR fluids by the AESM model is given. Moreover, by considering fully developed flows, analytic expressions for the conformation tensor **A** and velocity are derived.

The analytic solutions were found to be the same as given in [3], demonstrating that this novel algebraic model is capable of simulating shear flows defined by the classic differential FENE-CR model. In addition, a numerical methodology to solve the flow of a FENE-CR AESM fluid was implemented and applied to solve the flow in a two-dimensional channel. The analytic solutions were employed to verify the correctness and accuracy of the numerical methodology for simulating flows modelled by this new AESM and it presented a convergence order of 2. Future works include to apply this new algebraic extra-stress model to simulate flows in complex geometries and to test its abilities to predict viscoelastic flows at high Weissenberg numbers.

REFERENCES

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