Application of a Fast Fourier Transform Method to the Characterization of the Elastic Behavior of Trabecular Bone

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ABSTRACT

An alternative for using grafts or bone cement for the filling of bone cavities is the use of a bone scaffold that provides a temporary load-bearing function. Within the framework of the development of a computational tool for the design of bone scaffolds, the application of a Fast Fourier Transform Method (FFTM) to the homogenization of the elastic behavior of trabecular bone is presented in this work.

The FFTM, which was first introduced by Moulinec and Souquet [1], is based on the exact expression of the Green function for a linear elastic homogeneous material. The formulation of the method reduces the Green function to the Lippmann-Schwinger integral equation, which is solved iteratively. The rate of convergence of the iterative procedure is directly related to the stiffness contrast between the phases. The simplicity of the FFTM greatly reduces the effort for the model data preparation when compared to FEM.

The FFTM was implemented in C and programmed in parallel using OpenMP. The stress fields that result from the FFTM are post processed to compute the macroscopic homogenized elastic tensor by means of the asymptotic homogenization method. This post processing is performed using a Matlab subroutine.

The performance of the method is assessed for actual and artificial trabecular bone geometries. The geometries for the actual trabeculae are obtained from micro-tomography images. The images are segmented and binarized to generate the models. The results for the homogenized elastic tensors are compared to those obtained by Ibarra Pino and Cisilino [2], who also used the asymptotic homogenization method, but based on finite-element stress results. The geometries of the artificial trabeculae are retrieved from the database developed by Kowalczyk [3]. These are computer-generated trabeculae with their geometries locally described by a set of scalar parameters. Their macroscopic (continuum) elastic properties are assumed orthotropic and expressed as known functions of the geometric parameters.

The FFT method is found suitable to provide results of the same accuracy of FEA. To this end, guidelines are given for the model discretization and for the selection of the stiffness contrast between the void and bone phases. It is concluded that the FFT method can be effectively used for the homogenization of the elastic behavior of the trabecular structures with densities covering the range found in human bones.

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