

# Existence and stability of minimizers of mixed anisotropic $BV-L^2$ Tikhonov-Phillips functionals: applications to image restoration.

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## ABSTRACT

During the last two decades several generalizations of the traditional Tikhonov-Phillips regularization method for solving inverse ill-posed problems have been proposed. Many of these variants consists essentially in modifications of the penalizing term, which forces certain features in the obtained regularized solution ([2], [4]). If it is known that the regularity of the exact solution is inhomogeneous it is often desirable the use of mixed, spatially adaptive methods ([1], [3]). These methods are also highly suitable when the preservation of borders and edges is also an important issue, since they allow for the inclusion of anisotropic penalizers for border detection ([5]).

In this work, we propose the use of a penalizer resulting from the convex spatially-adaptive combinations of classic penalizing  $L^2$  and anisotropic bounded variation seminorm. Results on existence and uniqueness of minimizers of the corresponding Tikhonov-Phillips functional are presented. Stability results of those minimizers with respect to perturbations in the data, in the regularization parameter and in the operator are also established. Several applications to image restoration problems are showed.

## REFERENCES

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