

# An Harmonic 1D-Element for Nonlinear Analysis of Axisymmetric Structures: the Case of Hot Rolling

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## ABSTRACT

Harmonic finite elements have been developed [1] to deal with axisymmetric structures with non axisymmetric loads. Harmonic formulation is based on a Fourier series expansion of loads and displacements. This produces a set of linear systems of equations which, due to the orthogonality of Fourier terms, are uncoupled, thus giving a diagonal stiffness matrix with a significant reduction of the computational effort. In the nonlinear case, instead, the equations are coupled and therefore the harmonic approach seems not to be convenient. To overcome this difficulty in [2] a 2D mechanical element is developed to deal with an elasto-plastic analysis based on the initial stress method.

In this work an original 1D harmonic thermal and mechanical finite element is developed thus permitting a further reduction of computational time to be achieved, if the non-linear analysis is performed according to the initial stress method. The proposed formulation consists of a two-node element with linear shape function. The semi-bandwidth of the stiffness matrix is respectively one in the thermal analysis and three for the mechanical problem.

Hot rolling mill is a practical case that can take advantage of the proposed modeling approach. Work rolls in hot rolling mills are axisymmetric structures subjected to cyclic non axisymmetric thermal and mechanical loading. It is therefore necessary to perform numerical simulation of several revolutions until steady state condition is reached, taking also into account the elasto-plastic behaviour of the material. According to the literature, it seems reasonable to suppose that geometry and loads are constant along the roll axis and thus to adopt a 2D plain strain model. Nevertheless, even this 2D approach [3] requires an extremely long computational time, which allows only thermal loads to be considered in the mechanical analysis.

In this work a transient thermal and mechanical simulation of a work roll is performed. The localized mechanical loads, due to the interaction of the work roll with the strip material and with the back up roll are also taken into account. Even if a high number of Fourier terms is retained, this 1D harmonic formulation provides a strongly reduced computational time with respect to the 2D analysis.

## REFERENCES

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