First Pan American Congress on Computational Mechanics
PANACM 2015
April 27-29, 2015, Buenos Aires, Argentina

An axisymmetric DtN finite element method for problems in geophysics

Eduardo Godoy*, Mario Durán†

* INGMAT, José Miguel de la Barra 412, 4to piso, Santiago, Chile
eduardo.godoy@ingmat.com, www.ingmat.com

† Facultad de Ingeniería, Pontificia Universidad Católica de Chile, Av. Vicuña Mackenna 4860, Macul, Santiago, Chile
mduran@ing.puc.cl, www.ing.puc.cl

ABSTRACT

An important problem in geophysics is that of determining deformations and stresses around excavations in mining or civil engineering. To model mathematically this problem, a common simplification is to consider a semi-infinite elastic domain representing the solid earth. As such domain is unbounded, special numerical procedures need to be applied to solve this problem. The Dirichlet-to-Neumann (DtN) finite element method appears to be a powerful numerical technique to that purpose, since it uses the DtN map to obtain *exact* boundary conditions on an artificial boundary (of circular or spherical shape), which are combined with a finite element scheme in the bounded computational domain lying inside that boundary. This procedure has been successfully applied for different problems in infinite exterior domains, since in this case it is usually possible to compute an explicit closed-form expression for the DtN map. However, in the case of a semi-infinite elastic domain this is not, in general, possible. For this reason, the use of the DtN finite element method in geophysical applications has been rather limited.

In this work, we present a DtN finite element method for problems of elasticity in semi-infinite domains. The excavation is represented as a local perturbation on the plane surface, which is assumed to be axisymmetric about the vertical axis, allowing us to treat the whole elastic problem as axisymmetric, provided that only axisymmetric loading is present. The semi-infinite domain is truncated by introducing a semi-spherical artificial boundary surrounding the local perturbation. Then a finite element scheme is established in the resulting computational domain with exact boundary conditions in the artificial boundary. The DtN map is approximated numerically only in those boundary integral terms of the finite element formulation where it appears, in a similar way to what done in [2] for the two-dimensional case. To that purpose, a semi-analytical method is used, which is based on the procedure reported by Boccardo, Godoy and Durán in a very recent paper [1] to compute displacements and stresses in a isotropic elastic half-space with a hemispherical pit and subject to gravity. This procedure uses separation of variables to solve the problem in the residual semi-infinite domain lying outside the semi-spherical artificial boundary. The solution is expressed as a series with unknown coefficients, which are approximated by minimisation of a quadratic energy functional appropriately chosen, leading to a symmetric and positive definite linear system for a finite number of coefficients, which is efficiently solved by exploiting its particular block-structure. This procedure allows us to combine the exact DtN boundary conditions with the finite element scheme in an approximate but very effective way. The method is validated by comparing the obtained results with the semi-analytical solution given in [1], which is used as a benchmark. The relative error is computed and analysed for different mesh sizes.

REFERENCES

[1] V. Boccardo, E. Godoy and M. Durán, "An efficient semi-analytical method to compute displacements and stresses in an elastic half-space with a hemispherical pit", *Adv. Appl. Math. Mech.*, to appear (2015).

[2] D. Givoli and S. Vigdergauz, "Artificial boundary conditions for 2D problems in geophysics", *Comput. Methods Appl. Mech. Engrg.*, **110**, 87-101 (1993).