

# One Dimensional Fully Automatic hp-Adaptive Isogeometric Finite Element Method Package

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## ABSTRACT

The hp adaptive finite element (hp-FEM) method is the most advanced algorithm delivering exponential convergence rate of the numerical error with respect to the mesh size [1, 2]. The version of the hp-FEM algorithm proposed by the group of prof. Demkowicz [3] generates in a fully automatic mode a sequence of meshes delivering the exponential convergence. The idea of the fully automatic hp-FEM algorithm is based on the two grid paradigm, where the numerical problem is solved over the coarse mesh and the fine mesh, and the fine mesh is utilized as the reference solution for estimating the relative error of the coarse mesh solution. For each element from the coarse mesh the hp-FEM algorithm considers different refinement strategies, the combinations of h and p refinements, and the optimal strategy is selected based on the error decrease rate. The elements with the relative error larger than 33% of the maximum error are refined in this way. The algorithm proposed by Demkowicz in [3] utilized the hierarchical basis functions, locally  $C^{p-1}$  but only  $C^0$  on the interfaces between elements. In this paper we propose the one dimensional version of the hp-FEM algorithm with the B-spline basis functions. The resulting isogeometric hp-FEM code delivers solutions providing global  $C^{p-1}$  continuity of the solution [4, 5]. We also present a parallel multi-frontal direct solver for GPU delivering the coarse and fine mesh solutions in the  $O(\log N)$  time [6]. The convergence of the resulting isogeometric hp-FEM software is tested on the number of model problems.

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