A MPFA-Diamond Method Coupled to a Very Higher Order Multidimensional Upstream Scheme for the Solution of Two-Phase Flows in Heterogeneous and Non-Isotropic Petroleum Reservoirs

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ABSTRACT

Under certain simplifying assumptions [1], the two phase flow of oil and water in petroleum reservoirs can be described by an elliptic pressure equation and a non-linear hyperbolic saturation equation. In the present paper we use a Multi-Point Flux Approximation Method based on a Diamond type stencil (MPFA-D) [2] to discretize the diffusive term that characterizes the pressure equation and a very higher order multidimensional approximation (up to fourth order) [3] for the advective term of the saturation equation.

In the MPFA-D scheme the flux on each cell face is explicitly expressed by two cell-centered unknowns defined on the cells sharing that edge and two auxiliary unknowns defined at the two edge endpoints. As the scheme is cell-centered, vertex variables are expressed as weighted linear combinations of the neighboring cell-centered unknowns. In order to avoid spurious oscillations in the saturation equation, we use the recently proposed Multidimensional Optimal Order Detection (MOOD) strategy [4] as an alternative to the traditional a priori limitation procedures. In this method, intrinsically multidimensional, a first solution is evaluated using numerical fluxes, computed from an unlimited higher order local polynomial reconstruction obtained by least squares method. Through an iterative decremental process, the polynomial degree is reduced on the control volumes where the DMP (Discrete Maximum Principle) condition is violated.

In order to verify the accuracy and robustness of the full finite volume strategy, we solve some complex benchmark problems found in literature, including highly heterogeneous and anisotropic reservoirs [5].

The classical strategy of using a TPFA to discretize the diffusive terms and the First Order Upwind Method to approximate the advective terms of the flow equations usually leads to poor resolution of saturation fronts due to excessive numerical diffusion. On the other hand, the ability to handle arbitrary unstructured polygonal meshes and full permeability tensors of the MPFA-D method and the use of a very higher order finite volume with a multidimensional limiting procedure (MOOD) increases the capacity to handle complex geometries producing very accurate results, reducing excessive numerical diffusion at a competitive way.

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