Stochastic Models for Anomalous Diffusion Processes

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ABSTRACT

Diffusion processes that do not follow classical Gaussian diffusion are labelled in literature as anomalous. In the last years, Fractional Calculus is emerged to be a valuable tool to study these phenomena that result to be well modeled by space-, time- and space-time fractional diffusion equations. A self-similar stochastic process with stationary increment is derived for studying anomalous diffusion. The adopted approach follows the idea originally proposed to obtain the so-called generalized grey Brownian motion [1, 2]. In particular, the stochastic particle trajectory is stated to be driven by the fractional Brownian motion but, for any realization, it is multiplied by an independent random variable that is constant in space and time. The resulting probability density function for the particle displacement can be represented by an integral formula. It is shown that for an appropriate distribution of the multiplicative random variable, the one-point one-time probability density function for particle displacement is equal to the solution of the spatially symmetric space-time fractional diffusion equation [3] for some particular interval of the involved parameters. In fact, let $0<\alpha<2$ and $0<\beta<1$be the fractional derivative orders in space and time, respectively, the resulting Hurst exponent of the fractional Brownian motion is $H = \beta/\alpha < 1$ so that the spatial derivative order is restricted to $0 < \beta < \alpha < 2$. Due to the fractional Brownian motion, this class of stochastic processes is self-similar with stationary increments in nature and uniquely defined by the mean and the auto-covariance structure analogously to the Gaussian processes. Special cases are the time-fractional diffusion, the space-fractional diffusion and the classical Gaussian diffusion. The research aims to provide a stochastic modeling approach to describes anomalous diffusion observed for example in plasma turbulence [4, 5].

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