Numerical Solutions to Free- and Moving-Boundary Problems using Universal Meshes

Evan S. Gawlik*, Adrian J. Lew†

* Computational and Mathematical Engineering, Stanford University
Stanford, CA, USA 94305
egawlik@stanford.edu www.stanford.edu/~egawlik

† Mechanical Engineering, Stanford University
Stanford, CA, USA 94305
lewa@stanford.edu http://lavxm.stanford.edu/wiki/Adrian_Lew

ABSTRACT

We present a class of numerical methods for the solution of free- and moving-boundary problems. The methods make use of a universal mesh: a background mesh that contains the moving domain for all times and conforms to its geometry at all times by perturbing a small number of nodes in the neighborhood of the moving boundary. The methods are able to handle large domain deformations easily (a common difficulty faced by conventional deforming-mesh methods) while representing the geometry of the moving domain exactly (a challenging task for conventional fixed-mesh methods).

The framework enables the construction of methods with high order of accuracy. In particular, under suitable regularity assumptions on the exact solution and the moving domain's evolution, we prove that our method's spatial discretization error is of order $h^{r-1/2}$ when it is applied to parabolic moving boundary problems using finite element spaces consisting of continuous functions that are elementwise polynomials of degree at most $r-1$, $r \geq 2$, where h denotes the maximum diameter of an element of the universal mesh [2].

REFERENCES

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[2] E.S. Gawlik and A.J. Lew, "Unified Analysis of Finite Element Methods for Problems with Moving Boundaries", *Submitted*, (2014).