MHM Methods for time dependent propagation of electromagnetic waves.

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ABSTRACT

In this work, we are interested in the propagation of electromagnetic waves in complex media. More precisely, we would like to study time dependent wave propagation problems with strong multiscales features (possibly in space and time). In this context we would like to contribute in the design of innovative numerical methods particularly well-suited to the simulation of such problems. Indeed when a PDE model is approximated via classical finite element type method, it may suffer from a loss of accuracy when the solution presents multiscales features on coarse meshes. To address this problematic, we rely on the concept of multiscale basis functions that is one solution to allow for accuracy even on coarse meshes. These basis functions are defined via algebraic relations. Contrary to classical polynomial approximation, they render by themselves a part of the high-contrast features of the problem at hand. Recently, researchers at LNCC (Laboratório Nacional de Computação Científica, Brasil) have introduced a family of finite element methods [1]-[2], called Multiscale Hybrid-Mixed methods (MHM), which is particularly well adapted to be used in high-contrast or heterogeneous problems.. The algorithm rely on a two level discretization.

The basis function being computed at the second (finer) level allow for the reconstruction of the solution via the communication at the first (coarser) level on the squeleton of the mesh. This type of family has been firstly designed in the context of stationary problems, such as Darcy equations.

In this work, we propose to extend the use of this Finite Element family to time dependent wave propagation problems. The model problem relies on the time dependent Maxwell's equations. The continuity of the electric field is relaxed via the introduction of a Lagrange multiplier. The solutions are expressed on a basis computed at the second level that incorporates the heterogeneity of the problem via the resolution of a PDE. Several schemes are proposed from implicit to explicit time schemes and continuous finite elements to discontinuous ones for the space resolution of the local problem at the second level. We propose some results on the validity of the algorithm from both theoretical and numerical point of view and will present first numerical results in 2D.

REFERENCES

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