Solution of two-dimensional Shallow Water Equations by a localized Radial Basis Function collocation method

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ABSTRACT

A localized version of the Radial Basis Function (RBF) collocation method, initially developed in global formulation by Kansa [1] and implemented locally by Lee [2], is employed for solving the two-dimensional Shallow Water Equations (SWEs). The Newton-Raphson method is used to solve the resulting system of non-linear algebraic equations and a first-order implicit scheme is implemented for temporal discretisation. In order to obtain non-smooth functions which describe height and velocity at supercritical regimes, RBFs are used to approximate dependent variable products instead variable values by means of a local formulation in generalized finite difference fashion. Accurate solutions are obtained for sub- and trans-critical regimes in one-dimensional problems whose analytical solution are reported in [3]. For verifying two-dimensional spatial and temporal discretization, an oscillatory problem with analytical solution are solved to show the capability of the method for accurately solve 2-D problems. In general, excellent results are achieved regarding the fact that no upwinding scheme are employed when solving transcritical regime problems.

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