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Determination of Two Unknown Thermal Coefficients Through a Mushy Zone with a Convective Overspecified Boundary Condition

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ABSTRACT

We consider a semi-infinite material that is initially assumed to be liquid at its melting temperature which, without loss of generality, we assume at 0 C. At time $t=0$, a heat flux (characterized by the constant $q_0>0$) is imposed at $x=0$ and then solidification ensues, where three distinct regions can be distinguished (for a complete description of this mushy model see [1]):
H1) Liquid, at temperature 0 C, occupying the region $x>r(t)$;
H2) Solid, at temperature $T(x,t)<0$, occupying the region $0<x<s(t)$, with $s(t)\le r(t)$;
H3) Mushy zone, at temperature $T=0$, occupying the region $s(t)\le x\le r(t)$, with two assumptions on this structure which depends on two parameters $\gamma>0$, and $0<\varepsilon<1$.

We also consider an overspecified condition on the fixed face $x=0$ given by a convective boundary condition characterized by the constant $h_0>0$ [3]. We deal with six unknown thermal coefficients: $k>0$ (thermal conductivity), $c>0$ (specific heat), $\rho>0$ (mass density), $\ell>0$ (latent heat by unit of mass), and $\gamma>0$, and $0<\varepsilon<1$ the two parameters of the mushy region.

The goal of this paper is the simultaneous determination of two thermal coefficients among $\{k,c,\rho,\ell,\gamma,\varepsilon\}$ when the constants $q_0>0$ and $h_0>0$, and the boundary $s(t)$ are determined experimentally. We obtain explicit formulas in 15 different cases, and we also give necessary and sufficient conditions on data for the existence of a solution. These results complement [2].

REFERENCES

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