PDE-Constrained Optimization Using Hyper-Reduced-Order Models

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ABSTRACT

The high-cost associated with solving large-scale systems of equations prevents the use of high-performance computing codes in many-query settings such as routine analysis and design optimization. These settings can require thousands of high-dimensional runs for different values of design parameters.

Because they reduce the dimension of the underlying equations, projection-based reduced-order models (ROMs) are naturally suitable to be used in many-query contexts. When the underlying equations are nonlinear, however, an additional level of approximation is required to achieve speedups with the ROM. These approximations, referred to as hyper-reduction here, encompass methods such as the Discrete Empirical Interpolation Method (DEIM) [1] and the Gauss-Newton with Approximated Tensors (GNAT) [2,3] to name just a few.

Hyper-reduction defines a reduced integration domain, the reduced mesh [3], on which the computations are carried. Because only a portion of the state vectors is available in a hyper-reduced computation, challenges however arise when using hyper-reduced-order models in the context of design optimization.

In this work, a comprehensive approach is developed to enable the use of hyper-reduced-order models (HROMs) in optimization. The approach proceeds by defining surrogates for the objective and constraints that only rely on the information available online [4]. These surrogates are defined and trained in an offline phase. During that phase, the HROM is also generated so that a robust reduced model is available online. Greedy procedures are developed in this work to ensure the robustness of the HROM in the entire parameter space.

Two applications to computational fluid dynamics demonstrate the potential of the method to generate optimized designs while incurring significant speedups when compared to a design procedure that uses the underlying high-dimensional model. It is also shown that such a procedure is particularly suited for non-convex design optimization problems for which it is necessary to consider multiple starting points to avoid converging to a local optimum. The multi-start setting amortizes the offline cost of constructing the HROM by using it multiple times online.

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