ALP method for non-linear non-polynomial models

Jean-Frédéric Gerbeau, Damiano Lombardi, Elisa Schenone *

Classical model reduction methods (such as POD, RB) rely on a database of pre-computed solutions of the system of interest in the aim of building an optimal low dimensional global basis to represent the evolution in a certain region of the parameter space. These methods have some limitations when transport phenomena are involved or when the system depends upon distributed parameters (thus infinite dimensional). For the latter case, a representative database computation can be very expensive, and out-of-database solutions could be featured by instabilities.

The ALP method consists in defining a moving modal expansion to represent the solution of a non-linear Partial Differential Equation, without relying on a database of pre-computed solutions. The basis evolution is determined by asking that the modes are, at each time, eigenfunctions of a given linear self-adjoint (with compact inverse) operator, depending upon the problem solution (and thus on time).

Some methodological aspects are investigated.

First, a cheap optimization procedure can help in setting up an optimal operator for the solution representation. The potential of a Schroedinger-type operator is optimized to represent the initial datum at best.

In order to apply the method to non-linear non-polynomial models an Empirical Interpolation strategy is devised. Given a basis of fixed size, a set of (N_p) collocation points is chosen by means of a greedy algorithm to provide optimal interpolation properties.

Heuristically, the modes define a linear vector space of dimension N_M which evolves (with a non-linear dynamics) into a space of dimension N_p. The details of the discretization and the scalability of the method are commented.

Some examples of application are presented. First, the method is tested on wave propagation in heterogeneous media. Then, an application to reaction-diffusion equations in the context of cardiac electrophysiology is shown. The complexity and the speed-up of the Reduced Model are detailed.

*INRIA Paris-Rocquencourt, B.P. 105 Domaine de Voluceau, 78153 Le Chesnay, France.