# The construction of Multiscale Hybrid-Mixed (MHM) finite elements spaces using the object oriented computation library NeoPZ

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## ABSTRACT

The development of computational frameworks capable to capture complex interactions between several coupled phenomena that take place, simultaneously, at different spatial and temporal scales, is a major challenge nowadays, and a research topic of special interest. In this direction, several tools have been developed in recent years [1,2,3,4], using different approaches. Another new method has been proposed in the finite element context, under the name of MHM (Multiscale Hybrid-Mixed) [5]. It combines the homogenization of fluxes across macro element boundaries with a detailed representation of the solution in element interior parts. Some important properties hold for the MHM method: (a) local systems to be solved, favouring parallel strategies, (b) the jumps of the state variables on the interfaces can be used as error estimators for adaptive purposes, (c) locally conservation holds, and (d) higher order precision can be easily achieved.

The purpose of the present work is to show a systematic procedure for the construction of finite element spaces required for MHM simulations. Furthermore, when static condensation is applied to the MHM implementation, the application to typical test problems show the drastic reduction in computational effort. The implementation is performed within the scientific computational environment called NeoPZ (http://code.google.com/p/neopz). NeoPZ contains modular classes and methods that enables the implementation of *hp*-adaptive finite-element algorithms, and is suited for parallel computing.

## REFERENCES

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