A New Form of Finite Element Method for Plane Stress/Plane Strain Problems in Reinforced Concrete Considering Material Nonlinearity and Cracking

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ABSTRACT

This paper proposes a new numerical analysis procedure for plane stress/plane strain problems in reinforced concrete with consideration of material nonlinearity and cracking. The traditional finite element analysis (FEA), with its reliance on apparatus of matrix algebra, is an essentially linear procedure. To be able to use it for nonlinear problems, the loading has to be performed in small steps and iterative procedures used to distribute residual nodal forces on each step. These procedures generally have rather poor convergence, which further deteriorates after the events of fracturing or cracking in the concrete. The results are especially unreliable in the proximity of the ultimate load when the algorithm may become critically unstable, which is not necessarily equivalent to a physical failure.

Instead, it is proposed to formulate the problem of response modelling of reinforced concrete structures in simple tests for loadbearing capacity (constant preload + active load increasing to failure) as a non-step procedure going immediately to the target load. The method is formulated in terms of stresses because the cracking and yielding conditions are more readily formulated in these terms, and also to take full advantage of the easily solved equations of static equilibrium (Navier). Material nonlinearity of concrete, reinforcing steel, and imperfect bond between them is accounted for by the use of direct energy method, because the energy can be calculated by finite formulas for any value of stress regardless of nonlinearlity of the deformation curve. A variational principle of minimum complementary energy (modified Castigliano) is formulated and a numerical method developed for minimizing it on the domain of statically admissible stress fields (SASF).

For the construction of SASF for arbitrarily shaped structures, a variant of the finite element procedure is proposed. The finite element technique and matrix algebra are efficiently used for representation of the structure geometry and static equilibrium conditions only, which are linear in nature. (Large deformations are not considered in this paper.) This allows exclusion of "dependent" variables and formulation of the energy functional minimization problem in terms of the "fundamental" variables only, which in plane stress / plane strain problems are only one per node. Reactions in the external restraints are considered in this model as additional unknown components of the stressed state.

Deformation properties of concrete and steel are described in terms of non-associated work-hardening theory of plasticity, formulated such that the plastic strains have a potential. This enables the structure to be analyzed directly for the ultimate load, skipping the initial steps of loading, which are of little interest. The method satisfies the results of uniaxial compression and dilatancy tests on concrete. Cracking and fracturing in concrete is considered by imposing additional conditions on SASF, similarly to the method of discrete cracks in conventional finite element analysis of concrete structures.

Performance of different groups of numerical methods for the Castigliano functional minimization is examined and a special variant of first-order method of variable metric is developed to accommodate "leaps" on the path of descent caused by cracking. A methodology for ultimate load capacity analysis is introduced, based on solution of a double optimization (or maximin) problem in the SASF domain. This allows determining the failure load as part of the normal computational process, not as a disruption to this process, which should improve reliability and credibility of the ultimate load prediction.