Fast and faithful numerical methods for fractional differential equations and related nonlocal models on composite meshes

Hong Wang

[*] Department of Mathematics, University of South Carolina, Columbia, South Carolina 29208, USA
hwang@math.sc.edu

ABSTRACT

Fractional differential equations and other nonlocal models provide an appropriate description of problems that exhibit anomalous diffusion and long range interactions, which cannot be modelled properly by second-order differential equations. However, the numerical methods for these nonlocal models often generate dense or full coefficient matrices, which usually require computational work of $O(N^3)$ and memory of $O(N^2)$ for where N is the number of unknowns.

We develop fast and faithful numerical methods for these model problems fractional partial by carefully exploring the structure of the stiffness matrices, including some Toeplitz-like structures on each structured subdomain and some low-rank decomposition of the submatrices representing the interaction between different subdomains. These methods have computational cost of O(N log N) and memory of O(N) on equidistant and composite meshes, while retaining the same accuracy and approximation property of the underlying numerical methods. Numerical experiments show that to achieve the same accuracy, the method reduces the CPU time from more than 2 months consumed by a traditional method to 5.74 seconds for a problem with 36,000 grid points on a work station with 128GB memory. This shows the utility of the method.We will also address mathematical issues that are characteristic for nonlocal models.