Hybrid direct and iterative solvers for h refined grids with singularities

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ABSTRACT

This paper describes a hybrid direct and iterative solver for two and three dimensional h adaptive grids with point singularities. The point singularities are eliminated by using a sequential linear computational cost solver O(N) on CPU [1]. The remaining Schur complements are submitted to incomplete LU preconditioned conjugated gradient (ILUPCG) iterative solver. The approach is compared to the standard algorithm performing static condensation over the entire mesh and executing the ILUPCG algorithm on top of it. The hybrid solver is applied for two or three dimensional grids automatically h refined towards point or edge singularities. The automatic refinement is based on the relative error estimations between the coarse and fine mesh solutions [2], and the optimal refinements are selected using the projection based interpolation. The computational mesh is partitioned into sub-meshes with local point and edge singularities separated. This is done by using the following greedy algorithm

```
1 Create forest of n initial mesh element trees with lists_of_neighbors
     sorted according to number of neighbors
2 repeat
3    Find a pair v and w with maximum number of common edges
4    Create new root node r
5    Assign v and w as child nodes of r
6    Merge lists_of_neighbors of v and w to new list for r
7    Add new tree to the forest sorted according to vol
8 until minimum number of initial mesh elements in patches > threshold
```

The numerical experiments show that identification and elimination of point singularities both in two and three dimensions reduce significantly the number of iterations of ILUPCG solver.

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