PANACM 2015
Sergio Idehlson (Eds)

DYNAMIC PROGRAMMING FOR MEAN-FIELD TYPE CONTROL: A NUMERICAL STUDY WITH APPLICATIONS

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Key words: Stochastic Control, Risk assessment, Hamilton-Jacobi-Belman, PDE, FEM.

Abstract.

Mean-field games with an infinite number of players yield in the limit to a new type of stochastic control problem where the coefficients are function of the probability measure of the mean stochastic process.

Assuming that a PDF exists, the problem is standard for calculus of variations but non-standard for dynamic programming. Using derivatives with respect to measures as in P-L. Lions[1] we shall derive the Hamilton-Jacobi-Bellman equation for these problems.

We shall present two numerical applications of the method, one for a portfolio optimization and another for the systemic risk problem studied by J. Garnier and G. Papanicolaou [2]. The problem is to find the probability of the rare event when a system of N banks optimizing their assets switch from a stable business as usual state into a new bank crisis state.

The problem is modeled by the Fokker–Planck equation for the PDF of the state of the system of N banks in financial state x at time t when N is large and when all banks are governed by the same stochastic differential equation for which the equilibrium state is the mean of all N equilibria. This equation depends on a control which can drive the banks from a desired state to a non desired state.

Using freefem++[3], we have computed the most probable transition, among the rare events, that passes from the desired state (right hump in figure 1) to the undesired state of "crisis" (left hump) and the cost function gives the transition probability for this event. By minimizing this cost fuction we minimize the risk of transition to a global crisis of the system of Banks.

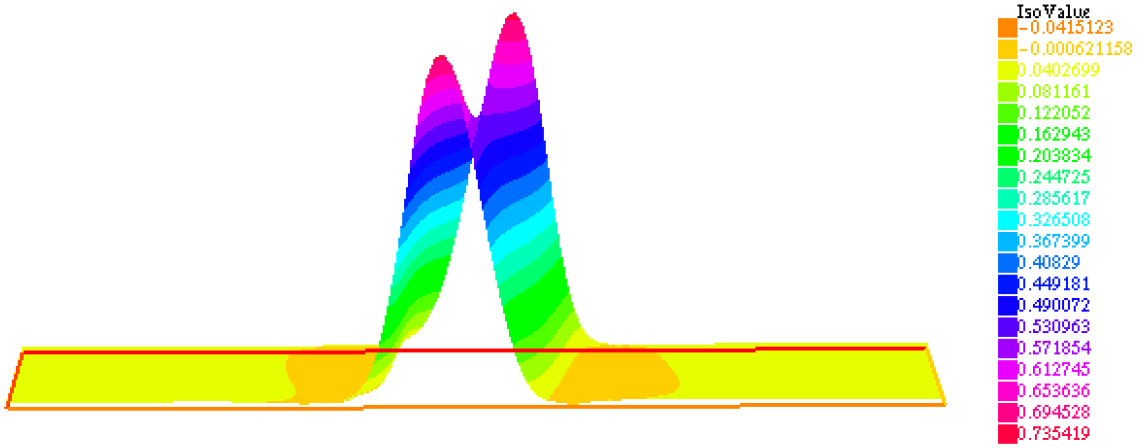

Figure 1: x,t → v, the PDF of the state of the system; -15<x<15 is horizontal, 0<t<2 is from front to back with origin at x=-15,t=0 on the lower left corner.

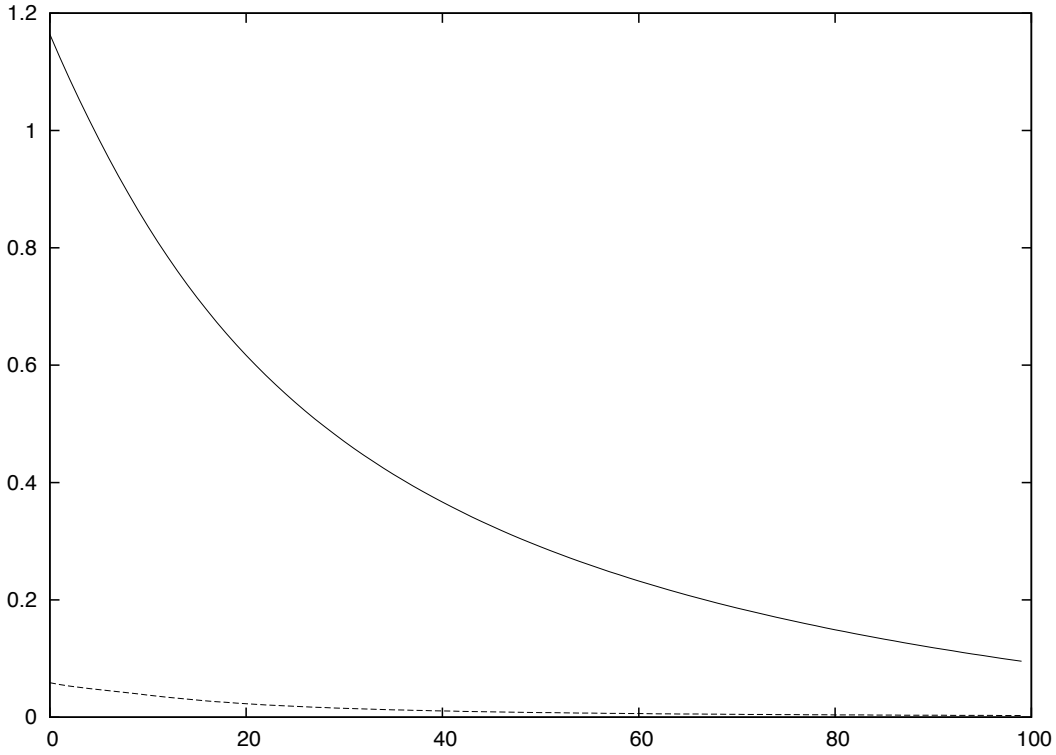

Figure 2: Iteration history: values of J (top curve) and \|grad_u J|^2 versus iteration count

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