Multi-Region Boundary Element Method and Tangent Operator Technique **Applied to Crack Propagation Modelling in Concrete Structures**

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ABSTRACT

The application of nonlinear fracture models to describe mechanically the crack growth phenomena in concrete structures has been performed by the scientific community recently. Among different models proposed in recent years, the cohesive crack model has demonstrated his efficiency and accuracy in simulating concrete cracking, as previously presented by [1-3]. In addition to that, the Boundary Element Method (BEM) is recognized as a robust and efficient numerical technique capable to address mechanical problems where the variables on analysis, stresses and displacements, for instance, exhibit higher gradients. These cases are currently observed in problems related to fracture mechanics domain. Moreover, the mesh reduction dimension provided by BEM makes remeshing procedure a less complex task.

In this regard, the present study aims at coupling the cohesive crack model with a numerical formulation based on the multi-region technique of BEM in order to simulate the mechanical behaviour of two-dimensional concrete structures subjected to initial cracks. Based on this coupling procedure, cohesive cracks are modelled along the interfaces of multi-region domains. It allows the representation of the structure nonlinear mechanical behaviour as well as its mechanical degradation process at the interfaces. The mechanical responses are determined using the algebraic BEM equations. In the implemented model, classical singular and hyper-singular BEM integral representations, considering isotropic bodies, were adopted.

The cohesive crack approach requires the choice of cohesive laws which relate interface tractions to crack opening displacements observed along each interface point. Taking into account that cohesive forces and crack opening displacements are interdependent, this problem is solved only in the context of nonlinear solutions. The nonlinear system of equations is solved, in this study, using a Tangent Operator scheme, where tangent prevision and tangent correction steps are required. Such scheme has demonstrated to assure better convergence and accuracy than the Classical Newton Raphson approach as the derivate set of algebraic equations incorporates the cohesive laws [4, 5]. The deduction of the tangent operator terms for each cohesive law applied is the main contribution of this study.

Three applications involving crack propagation in concrete structures were considered to validate the proposed formulation [1-3]. The first application is a three-point bended beam subjected to mode I of fracture. The second and third applications concern beams subjected to crack growth in mixed mode cases. A four-point shear beam with single notch and a four-point shear beam with double notch, respectively, were analysed. The results achieved by the implemented nonlinear approach were compared with numerical and experimental responses available in literature. Good agreement among these results indicates the accuracy and robustness of the proposed nonlinear scheme.

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