On a mixed least-squares formulation using different approximation spaces

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ABSTRACT

In the present work a mixed finite element based on a least-squares formulation is proposed. Here, the solution variables (displacements and stresses) are interpolated using different approximation spaces. Basis for the formulations is a weak form resulting from the minimization of a least-squares functional, compare e.g. [1]. In detail we compare the formulation with the stresses approximated in $H^1(B)$ or in $H(\mathrm{div},B)$, whereas the displacements are always interpolated in $H^1(B)$. For the conforming discretization of the Sobolev space $H(\mathrm{div},B)$ vector-valued Raviart-Thomas interpolation functions, see also [2], are used. As suitable functions for $H^1(B)$ standard interpolation polynomials of Lagrangian type are chosen. The resulting elements are named as P_mP_k and RT_mP_k. Here m (stresses) and k (displacements) denote the approximation order of the particular interpolation function. Due to the known drawback of weak performance especially for lower-order elements, see also [3], the presented interpolation setups are compared with respect to their performance and the size of the resulting systems of equations under cosideration of different orders of m and k.

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