Comparison of accuracy and computational cost of numerical methods for solving the Boltzmann Equation

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ABSTRACT

In this work three different numerical methods for solving the Boltzmann Equation will be compared: A deterministic kinetic solver based on the Spectral-Lagrangian method developed by Gamba et al. [1], a Boltzmann solver by Varghese et al. based on the Discrete Velocity Method [2] and the direct simulation Monte Carlo (DSMC) method of Bird [3]. An assessment is made on relative computational cost, both in terms of memory usage and CPU runtime to obtain a pre-set level of accuracy in the macroscopic moments, i.e. density, hydrodynamic velocity, stress, heat flux, etc. Due to the different nature of the three methods (deterministic vs. stochastic), special metrics for assessing the accuracy an dallowing a fair comparison will have to be defined.

The first test case chosen for this comparison consists of homogeneous relaxation towards equilibrium of a mixture of monatomic gases initially at different Maxwell distributions. This will permit to directly compare the collision routines of the methods without involving transport/advection phenomena. In a second test case, a steady normal shock wave (in the shock frame of reference) in a single-species monoatomic gas, such as argon will be simulated. The pre- and post-shock conditions are imposed as steady boundary conditions at the limits of a one-dimensional physical domain, whereas velocity space is treated as three-dimensional. For the deterministic solvers the pre-and post-shock gas states impose heavy demands on the resolution and extension of the velocity mesh, which directly affect the computational cost. In case of the DSMC method, statistical noise of the solution at high temperatures and low stream velocities is of concern.

REFERENCES

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