

# Universal Meshes: High-order simulation of problems with evolving geometries

**Adrian J. Lew** †

\* Stanford University  
Stanford, CA 94305-4040  
lewa@stanford.edu

## ABSTRACT

Multiple problems in engineering involve geometries that evolve with the problem. Fluid-structure interaction, phase transformation, and shape optimization problems are the most common, but crack propagation and solids undergoing extreme deformations problems need similar strategies as well.

Three types of approaches are typically adopted for these problems: periodic remeshing, arbitrary Lagrangian-Eulerian kinematic descriptions (ALE), and embedded or immersed boundary methods. The first one is generally considered computationally expensive, the second one breaks down under very large deformations, and the last one often leads to low-order methods because of a poor representation of the geometry.

In this talk, I will introduce the concept of a "Universal Mesh", which combines the best of each one of the above strategies. In a nutshell, a Universal Mesh for a class of domains is a triangulation that is able to mesh any of the domains in the class upon minor perturbations of the positions of its nodes. Hence, as the domain evolves, the perturbed Universal Mesh provides an exact triangulation of the geometry. It is then possible to formulate high-order methods for problems with evolving geometries in a standard way.

We have proved that, under certain conditions, the map that deforms the Universal Mesh to conform to the domain as it evolves defines a homeomorphism. These conditions impose restrictions on what constitutes a Universal Mesh. Additionally, for time-dependent problems, we have formulated a class of time integration methods for problems with evolving domains (such as Stefan's problem), and obtained conditions on the discretization scheme to render a high-order method. These conditions can be used to analyze the convergence of both ALE- and Universal Meshes-based schemes.

I will show applications of these ideas to fluid-structure interaction and crack propagation problems.