On numerical quadrature for C^1 quadratic Powell-Sabin 6-split macro-triangles

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ABSTRACT

The quadrature rule of Hammer and Stroud [1] for cubic polynomials has been shown to be exact for a larger space of functions, namely the C^1 cubic Clough-Tocher spline space over a macro-triangle if and only if the split-point is the barycentre of the macro-triangle [2]. We continue the study of quadrature rules for spline spaces over macro-triangles, now focusing on the case of C^1 quadratic Powell-Sabin 6-split macro-triangles. We show that the 3-node Gaussian quadrature(s) for quadratics can be generalised to the C^1 quadratic Powell-Sabin 6-split spline space over a macro-triangle for a two-parameter family of inner split-points, not just the barycentre as in [2]. The choice of the inner split-point uniquely determines the positions of the edge split-points such that the whole spline space is integrated exactly by a corresponding polynomial quadrature. Consequently, the number of quadrature points needed to exactly integrate this special spline space reduces from twelve to three.

For the inner split-point at the barycentre, we prove that the two 3-node quadratic polynomial quadratures of Hammer and Stroud exactly integrate also the C^1 quadratic Powell-Sabin spline space if and only if the edge split-points are at their respective edge midpoints. For other positions of the inner and edge split-points we provide numerical examples showing that three nodes suffice to integrate the space exactly, but a full classification and a closed-form solution in the generic case remain elusive.

REFERENCES

[1] Hammer, P. and Stroud, A. Numerical integration over simplexes. *Mathematical tables and other aids to computation*, 10(55):5137-139, 1956.

[2] M. Bartoň and J. Kosinka. Gaussian quadrature for C^1 cubic Clough-Tocher macro-triangles. *Journal of Computational and Applied Mathematics*, 349:239–250, 2018.