An Adaptive Volumetric Subdivision Method for the Solution of Partial Differential Equations

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ABSTRACT

Subdivision methods have been widely used in the field of computer graphics to generate smooth surface representations. More recently, they have been used in solvers for partial differential equations in 2D or on smooth surfaces in the framework of finite elements [1] or isogeometric analysis [2]. The resulting analysis mesh benefits from increased continuity and has been used for solving problems involving 4th order or higher partial differential equations. The available theory for subdivision volumes is however much more limited.

In this study, we present a volumetric subdivision method based on the Catmull-Clark subdivision algorithm. In the proposed method, the mesh is formed by hexahedral elements, and the boundary of the solid mesh is a Catmull-Clark surface. Corners, creases and other sharp features can be considered by a selective smoothing algorithm to allow flexible free form modeling. A crucial point is the extension of the idea of limit surface to 3D, where we implement it for evaluation at points near extraordinary vertices and edges. We also consider adaptive and multi-resolution schemes to provide local refinement in the regions where the solution or the geometric features are non-smooth. The efficiency and flexibility of the proposed method is analyzed by applications to benchmark problems of engineering interest.

REFERENCES

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